Summary:

On the firefighting boom of a MI 17 helicopter, an experimental modal analysis has been carried out in four different configurations. The results revealed significant differences to the analytical data. The second vertical bending mode of the boom was detected to be in resonance with the downwash forces of the propellers. In order to predict the excitation of the boom in flight, the analytical model was updated. All four configurations were modeled in FEM. Since parameter variation affected all four models, a so called “multi-model updating” technique was used. After additional flight tests without boom and ground tests with boom, it was possible to assess the relevance of the resonant situation. In the final operational deflection shape test with boom the assumptions were verified during flight. Finally, the acting dynamic downwash forces were updated using the ODS data.

Though the structure is not very complex, the analytical model revealed significant errors leading to a hazardous mis-assessment. Thus, the boom serves as a striking example how methodic model updating based on various test data is necessary to produce validated models and reliable assessment.

Keywords:

Multi-Model Updating MMU, Force Identification, Experimental Modal Analysis, Operational Deflection Shape Analysis ODS.
1 Introduction

Kuala Lumpur is one of the vastly growing East-Asian cities with high rising buildings, among them the tallest buildings in the world, the Petronas Towers. With the development of the tall edifices also the risk of local fire in upper floors increases which might be difficult to access rapidly by fire fighting personnel. To enhance a first and quick response at a critical stage of a fire, the firefighting department BOMBA of Kuala Lumpur is equipped with firefighting helicopters. Comparable helicopters are in service in the US and in Japan. The latter are equipped with a boom in flight direction. The concept developed by AIRROD Malaysia, however, provides a lateral cantilever boom mounted perpendicular to the helicopter fuselage. This allows better maneuvering in case of sudden critical incidents like explosions. The boom itself consists of an aluminum girder with three pipe shaped main bars that are stiffened by x-braces (fig. 1). The lower pipe is simultaneously used as a water pipe with a nozzle at the tip. The boom itself can be driven by a hydraulic device to move it to a parallel position along the helicopter fuselage. In this position, another attachment is provided next to the pilots compartment. The clamp can be operated by the pilot.

Figure 1: Top view of the Boom in Operating (Perpendicular) and Maneuvering (Parallel) Position, Replacement of the Rigid Boundary Conditions by Stiff Spring Elements.

Preliminary numerical analyses have been performed to investigate the dynamic behavior of the boom [1] in four different configurations:
- Boom in operating position (perpendicular), empty.
- Boom in operating position (perpendicular), pipe filled with water.
- Boom in maneuvering position (parallel, clamped), empty.
- Boom in maneuvering position (parallel, clamped), pipe filled with water.

The mass of the water in the pipes increases the total weight of the boom \( m = 57 \text{ kg} \) by 40 %. The dynamic behavior is significantly changed.

The safety of the boom has been verified for the following load cases:
- Static load of the inertial forces of the boom itself.
- Static load by downwash during hovering.
- Static load by wind on maneuvers.
- Dynamic load by downwash during hovering.

The assessments of the last load case were mainly based on the prediction of the modes of the boom. The main dynamic forces are the downwash forces excited by the rotor blades at 16.2 Hz (rotation speed: 196 rpm x 5 rotor blades). No critical eigenfrequency was found in the vicinity of this excitation frequency.

The task of the following investigations by Mueller-BBM initially was to validate the analytical results by an Experimental Modal Analysis (EMA) and an Operational Deflection Shape analysis (ODS).

2 Experimental Modal Analysis (EMA)

The experimental modal tests were performed for the four different configurations. In addition to the analytical configurations, also the situation was considered when the boom is not yet attached to the clamp in the parallel position.

The boom was excited at the tip with an impact hammer. At up to 8 testing points the acceleration response was measured simultaneously at a maximum of 15 channels in vertical and horizontal
direction (fig 2a). For data acquisition, a multi-channel VXI hardware in connection with the PAK data acquisition software was used [7].

Exploiting the response data, the curve fit revealed mode shapes and eigenfrequencies in the range from $f = 16.2$ Hz (parallel with water) up to 21 Hz (perpendicular w/o. water) for the second vertical bending mode of the boom (fig. 2b). Thus, the fundamental assumption of the analytical investigation that no resonance occurs near the excitation frequency of the downwash forces was obsolete.

3 Model Updating

3.1 Substitution of the Boundary Conditions

The reason for the discrepancy of the original analytical results and the testing data has been located in the stiffness of the brackets (fig. 3). By lack of data, the original analytical model assumed the attachments to the helicopter to be rigid. As was shown by the experiment, this assumption was approximately true for the operating condition but not at all for parallel position. To assess the situation, a valid model was required.

With the boom detached, the horizontal and vertical impedances at the brackets on the helicopter fuselage were determined experimentally. The lower impedances which are more significant for the eigenfrequencies revealed approximately stiffness characteristics. Therefore, it was appropriate to replace the rigid boundary conditions by stiff springs (fig. 1). Hence, a much better correlation was achieved, but still better accordance was desired especially near resonance. Instead of detailed modeling of all the fuselage of the helicopter, updating of the boundary stiffnesses seemed to be the more appropriate way to attain satisfactory correlation.

3.2 General Approach

For further tuning of the model, the widely used sensitivity based updating technique as described for instance in [2] was used. The main steps shall briefly be summarized.

The errors in the responses e.g. eigenfrequencies are defined as residuals of a non-linear optimization algorithm. The standard approach adopted in model updating is to linearize the estimation problem about the current parameter estimate, and to iterate until convergence.

Relating the necessary parameter update $\Delta P$ with the error in the response $\Delta R$, a gradient sensitivity matrix $S$ is computed (1).

$$\begin{bmatrix} S \end{bmatrix} = S_{ij} = \begin{bmatrix} \delta R_i \\ \delta P_j \end{bmatrix}$$
For a series of parameter-response combinations, closed formulas exist so that expensive system solving with varying parameters is avoided. Eq. 2, for instance, represents the sensitivity of an eigenfrequency.

\[
\frac{\delta f_i}{\delta P_j} = \frac{\{\psi_i\}^T\left(\frac{\delta[K]}{\delta P_j} - 4\pi^2 f_i^2 \frac{\delta[M]}{\delta P_j}\right)\{\psi_i\}}{8\pi^2 f_i \{\psi_i\}^T[M]\{\psi_i\}}
\]  

(2)

To solve the equation for the parameter update \(\Delta P\), the sensitivity matrix must be inversed. Since the number of parameters rarely corresponds to the number of responses, the sensitivity matrix is not quadratic. For the inversion, a least square method is used to create a generalized inverse. However, there are a number of issues relating to the conditioning of the estimation problem that must be addressed. To overcome these problems, weighting is applied to both the measured data and the parameters with the following consequences:

- Ill-conditioning of the inversed sensitivity matrix can be controlled.
- Thus, different classes of responses, e.g. eigenfrequencies and MAC-values, can be treated simultaneously.
- The optimization algorithm can be regarded as a Bayesian parameter estimation algorithm where the weighting matrices are related to known or estimated standard deviations of the parameters and responses.

Since the \(\Delta R\) is not differential and the relation between parameters and responses is non-linear, the optimization has to be performed iteratively. A new response with the updated database is computed and compared to reference. The next parameter update is computed based on a new sensitivity matrix. The loop is repeated until satisfactory convergence is achieved.

### 3.3 Multi-Model Updating

In difference to the classical updating procedure where the responses of one model are used as residual functions, here the results of four different configurations had to be taken into account. The fact had to be considered that a parameter change causes a different response in all four analytical models. Therefore, the method is extended to a so-called multi-model updating technique [3].

A global sensitivity matrix (and weighting matrix) is assembled of submatrices descending from the individual models. As the parameters are the same in all four models, the number of rows of the sensitivity matrix is fixed whereas the number of columns is increased by the different responses of each model.

In the present investigation, the error of the eigenfrequencies of the 2nd, 3rd and the 4th mode were defined as residual functions. The 4th eigenfrequency was weighted higher in order to focus the algorithm on the reduction of the error at the second vertical bending model.

The boundary stiffnesses of the brackets \(k_x(2), k_y(2)\) and \(k_z(1)\) as well as the longitudinal stiffnesses of the hydraulic device \(k_{II}(2)\) were defined as parameters. Scaled with higher confidence also global parameters like the Young’s modulus “E”, the density of the material “rho”, the pipe diameter “Diam”, the nozzle mass “m” and the mass of the water “H20” were chosen. Fig. 4 shows the complete sensitivity matrix.

It should be noted that of course the mass of the water was only considered simultaneously in the two corresponding models as indicated in fig. 4. The longitudinal stiffnesses of the hydraulic device were allowed to be different in the perpendicular and parallel position and thus required separate treatment.

![Multi-Model Sensitivity Matrix](image-url)
Before discussing the results, it should be noted that the success of the updating depends on the choice of the parameters and responses. Furthermore, weighting plays an important role. By consequence, there is no unique solution to the problem. However, this should not be considered as a drawback of the procedure but rather as an opportunity for the engineer to explore the analytical model and its performance.

In this example, a sequence of combinations has been examined to enhance the correlation without losing too much of the physical background of the structure. For this purpose, a software tool is required where the possibility of simultaneous examination and correlation of a test database and an analytical database is provided. Since the results produced by different engineering software have to be integrated, up-to-date interfaces have to exist. Next, a powerful graphical user interface is required to visualize the complex arithmetic.

The core of the described model updating procedure is the computing of the sensitivities. The software must therefore supply a library of closed formulas for all parameter-response combinations of the database.

Finally, the database and the standard routines like sensitivity computation must be accessible for manipulation. For the multi-model updating it was necessary that the standard procedure was customized to access more than one project file within the algorithm.

With FEMtools [8] a specialized software for the integration of test and analysis was used which fulfills these requirements.

### 3.4 Updating Results

The updating results are summarized in fig. 5. Starting from an error average of more than 50% in the 2nd, 3rd and 4th eigenfrequency, the error was reduced to 15% simply by the introduction of stiff boundaries. By updating, the error was further decreased to less than 5% for all 4 models or less than 1 Hz in an absolute scale. For the most critical configuration in parallel position with water the error is less than 1% at 16.2 Hz.

As a side effect, also the correlation between the experimental and analytical shape functions was improved significantly. In fig. 6, the MAC matrices for the corresponding models are depicted. In the matrix, each analytical eigenvector \( \Psi_a \) is paired with each experimental eigenvector \( \Psi_e \). A high value signifies good correlation.
Initial model, fixed boundaries

Updated models: perpend. / perpend. water / parallel / parallel water

Figure 6: MAC-Matrices: Correlation between Experimental and Analysis Eigenvectors, Comparison between Initial and Updated Models.

Figure 7: Mode Shape Pair at 16.2 Hz (Parallel, Water).

Fig. 7 shows the mode shape pair of the 2nd vertical bending mode in parallel position with water at \( f = 16.2 \) Hz.

Generally, the model would now be fit enough for further analytical investigations like e.g. stress analysis. Since the prediction of the response in flight condition was to be compared with Operational Deflection Shape testing (ODS), the model was further tuned to match also the Frequency Response Functions (FRF) of the experimental modal analysis.

When defining all existing measured FRF functions as residuals, one realizes that a lot of constraints have to be regarded. In addition, the sensitivities vary enormously near resonance. Therefore, a so-called Cross-Signature-Assurance Criterion (CSAC) is introduced as an error function (fig. 8) instead.

Figure 8: Frequency Spectrum of the CSAC-Criterion (Continuous Line): Correlation between Experimental and Analytical FRF-Data.
The CSAC is computed in analogy to the MAC for each paired FRF at each discrete frequency step. A high value between 0 and 1 indicates good correlation between a series of paired FRF functions. The CSAC is less sensitive in the vicinity of resonance peaks. As result, it was also possible to match the FRF in a satisfactory way (figs. 9) especially around the critical frequency range of $f = 16.2$ Hz.

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Figures 9: FRF-Pairs, Analytical (Continuous) and Test (Stepped).

4 Flight Tests, Operational Deflection Shapes

After several investigation steps including stress analysis, ground test, flight test without boom, it was possible to predict that the resonance in the second bending mode was not critical compared with other load cases. One reason was that the 2nd bending mode of a cantilever beam is antimetric whereas the downwash forces can be assumed to be uniformly distributed.

Nevertheless, it was possible to show that due to the resonant excitation the response of the boom was significantly higher in parallel position with water. Over a time window of 1200 s, the acceleration response of the boom at 16.5 Hz as well as at 19.25 Hz (resonance in operating position) is traced in fig. 10. The resonance peak is shifted from 19.25 Hz to 16.5 Hz and the amplitude is increased by a factor of 5 to 6 while moving the boom from perpendicular into parallel position.

Fig. 10b depicts the overlayed APS spectra of acceleration at $t = 816$ s. The peak at $f = 16.2$ Hz is clearly visible.

5 Force Identification

For further analysis on modified structures it was desirable to have a validated load model. Since the downwash forces were difficult to measure, indirect identification of the forces or updating of approximate estimates of the operating forces was a practical alternative [5], [6].

The original estimated load vectors $F_{e,i}$ are scaled by an unknown factor $\alpha_i$ according to eq. 3.

$$\{F_{e}\} = \sum_{i=1}^{N} \alpha_i \{F_e\}$$

To determine the scaling factor $\alpha_i$ the equation to compute displacement response vector $\{X\}$ in the frequency range spanning N modal parameters is inversed:

$$\{X\} = \sum_{i=1}^{N} \frac{\{\Psi\} \{\Psi\}^T \{F_e\}}{(\lambda_i^2 - \omega^2)}$$

![Figure 10a: Time Tracks of the Amplitudes at $f = 16.5$ Hz and 19.125 Hz at the Tip of the Boom.](image)

![Figure 10b: APS of Acceleration at $t = 816$ s.](image)
\( \lambda^2 \) is the \( i^{th} \) eigenvalue of the damped system, \( \omega^2 \) the excitation frequency and \( \{\Psi\} \) are the normal modes of the structure.

Since the number of DOF of the experimentally determined displacement vector \( \{X\} \) is much smaller than the number of DOF of the analytical model, a system reduction method like Guyan reduction or the System Equivalent Reduction Expansion Method SEREP is applied. The result of the force updating is shown in fig. 11 for the vertical direction.

Figure 11: Identified Vertical Dynamic Forces at \( f = 16.2 \text{ Hz} \), Side View.

Applying the updated force vector in eq. 4 again an analytical displacement vector can be compared to the test results of the ODS. In analogy to the MAC a Displacement Assurance Criterion (DAC) is computed to get a quantitative correlation indicator.

For the test displacement shape in fig. 12, a correlation factor of DAC = 75\% was attained. Considering the only approximate stationary condition, this is an acceptable result.

Figure 12: ODS at \( t = 816 \text{ s} \) and \( f = 16.2 \text{ Hz} \), Deformed and Undeformed, Side View.

6 Conclusions

- The investigation of the helicopter boom served as an example to show the variety and the efficiency of model updating techniques.
- The feedback from the customer and in this case also from the pilot proved that throughout the validation of the model a high confidence could be established in this safety-relevant concern.
- The fact that during the flight tests the responsible engineer had to be on board gave the demand of quality management and validation also a very personal aspect.
- Regarding the tight time constraints, it was indispensable to work with professional and validated specialized software that assures a reliable and adaptable environment.

7 References

